Borel sets in Baire space and types of theories

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Let $\mathbb{A}$ be a first-order structure of a countable language L, X a countable subset of A, $L_X = L \cup \{\underline{a} \mid a \in X\}$ and $\mathbb{A}_X$ the simple expansion of $\mathbb{A}$ to L_X. We consider two topologies defined on the set of valuations over domain A.

The first one is the Cantor space and it is related to the map $*$ from Sent_{L_X}, the set of sentences of L_X in logic $\mathcal{L}_{\omega_1\omega}$, into the set of infinitary propositional formulas of $\mathcal{L}_{\omega_1}$ with appropriately chosen set $\mathcal{P}$ of propositional letters. The map $*$ allows us to code various problems on the model $\mathbb{A}$ by infinitary propositional sentences. In particular, we show in a uniform way, considering a countable expansions L' of L, that for various counting functions concerning the model $\mathbb{A}$ CH holds. Examples are Aut$\mathbb{A}$ (Kueker), End$\mathbb{A}$ and Con$\mathbb{A}$ (Burris and Kwaitinetz), and other problems on infinitary graphs (as coloring), and certain types of coverings of the plane.

The second one is the Baire space defined on the set of all valuations of predicate formulas. Appropriate functions related to formulas and types with finitely many free variables are continuous, even in the infinitary logic $\mathcal{L}_{\omega_1\omega_1}$. If they have countable many free variables, the related subsets of the space are Borel. Therefore, CH holds also for various counting functions concerning the model $\mathbb{A}$.